Lecture 6 - Relative velocity, CAPA problems

Relative velocity

An example (angle problem in Set 2): It is a dark and stormy night and you are driving through the rain in your car. Prior to getting in your car you noticed that there was no wind so the rain was falling vertically. However, when you are in your car and driving at a constant speed of $50mph$, you notice that the rain is hitting your windscreen at an average angle of 25^0 to the horizontal. Find the terminal velocity of the raindrops. This is a typical problem in the analysis of relative velocity. We have to understand how the velocity of the rain changes when we are in one reference frame (the car moving at $50mph$) as compared to another reference frame (the earth, ie someone standing stationary on the earth). To understand relative velocity problems it is essential to be systematic, as they can be pretty confusing. Here is a good way to proceed.

(i) Define two reference frames, in our case the earth (E) and the car (C). These two reference frames are moving at constant speed with respect to each other $\vec{v}_{CE} = 50mph$. The subscripts mean the velocity of the car (C) with respect to, or as measured by, someone standing on the earth (E).

(ii) Define the velocity of the moving object in these two reference frames. In our case the object is a raindrop. We define the velocity of the raindrop with respect to earth $\vec{v}_{RE}$ and we also define the velocity of the raindrop with respect to the car $\vec{v}_{RC}$.

(iii) Write the relationship between the three velocities, in our case the relationship is,

$$\vec{v}_{RE} = \vec{v}_{RC} + \vec{v}_{CE} \tag{1}$$

To get started in these problems you need to choose what the two reference frames are and what the object is. In problems where a boat travels in the water it is usually best to define the two reference frames as earth (E) and the water(W), while the object is the boat(B). In problems where a plane travels through the air, usually it is best to treat the earth (ground or stationary air) (E) and the moving air (A) as the two reference frames and the plane (P) as the object. After that you can follow steps (i) - (iii) and use an equation like Eq. (1).

Note that if you change the order of the subscripts, the sign of the velocity changes, for example $\vec{v}_{RE} = -\vec{v}_{ER}$. This makes physical sense as the velocity of the rain viewed from Earth is downward, but if we are sitting on the raindrop, it would look as though the earth were rushing up to meet us. The procedure above can be adapted to all of the relative velocity problems which you will need to solve.

Lets now figure out this particular problem. To do that we draw a triangle representing the vector addition of Eq. (1). Since in our problem $\vec{v}_{RE}$ is vertical and $\vec{v}_{CE}$ is horizontal, the triangle is a right angle triangle. From this triangle we can use trigonometry to find the magnitude $v_{RE} = 50Tan(25^0) = 23mph$.

We will also go through some of the problems in Set 2. Hint: planeinwindb.problem is quite hard. You need to use the Cosine rule given on the formula sheet and you need to figure out what the A,B,C etc are in this application. We will go through that in detail in class.